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$$\begin{array}{lccc} \text{for } a, & + & + & + \\ \text{for } b, & + & - & - \\ \text{for } c, & - & + & - \\ \text{for } d, & - & - & + \end{array}$$

For the z_1, z_2, z_3 in this solution III must be substituted the roots of the cubic II.

Example. As an example take the biquadratic

$$x^4-x^3-7x^2+x+6=0.$$

Here we have,

$$a_1=-1, \quad a_3=1,$$
$$a_2=-7, \quad a_4=6,$$

from which the cubic becomes $z^3+7z^2-25z-175=0$, of which the roots are 5, -7, and -5. Thus the roots of the biquadratic are

$$\tfrac{1}{2}\{1\pm\sqrt{1-2\{-7\pm1\pm5\pm1\}},$$

or 1, -1, -2, 3, which are seen to be correct.

Care must be exercised that the proper sign before the main radical is taken.

Urbana, Ill., October 9, 1897.

EQUATION OF PAYMENTS.

By J. A. CALDERHEAD, A. B., Professor of Mathematics, Curry University, Pittsburg, Pennsylvania.

Let it be required to find the equated time of two payments, P and P_1, due at the end of t and t_1 years respectively, and r being the rate of interest.

Represent the equated time by x when $t>t_1$.

I. By Simple Interest.

1st Method. The discount on P for $(t-x)$ years must equal the interest on P_1 for $(x-t_1)$ years.

$\dfrac{P(t-x)r}{1+(t-x)r}$=discount on P due $(t-x)$ years hence.

$P_1(x-t_1)r$=interest on P_1 for $(x-t_1)$ years.

$$\therefore \frac{P(t-x)r}{1+(t-x)r}=P_1(x-t_1)r.$$

$$\therefore\ x=\frac{1}{P_1r}(P+P_1+P_1rt+P_1rt_1$$

$$\pm\sqrt{P^2+P_1^2+P_1^2r^2t^2+P_1^2r^2t_1^2+2PP_1+PP_1rt+2PP_1rt_1+P_1^2rt})\ \ldots\ldots\ldots(1).$$

2nd Method.

$\frac{P}{1+rt}$=present worth of P due t years hence.

$\frac{P_1}{1+rt_1}$=present worth of P_1 due t_1 years hence.

$\frac{P+P_1}{1+rx}$=present worth of $P+P_1$ due x years hence.

$\therefore$ Suppose $\frac{P}{1+rt}+\frac{P_1}{1+rt_1}=\frac{P+P_1}{1+rx}$;

then $x=\frac{Pt+P_1t_1+Prtt_1+P_1rtt_1}{P+P_1+Prt_1+P_1rt}$ ....(2).

Since (2) differs from (1), the sum of the present worths of P and P_1 due in t and t_1 years respectively, at simple interest, is not equal to the present worth of $P+P_1$ due at the equated time; hence, the second method is not correct when we compute by simple interest.

II. BY COMPOUND INTEREST.

1st Method.

$P\left[1-\frac{1}{(1+r)^{t-x}}\right]$=discount on P for $(t-x)$ years.

$P_1[(1+r)^{x-t_1}-1]$=interest on P_1 for $(x-t_1)$ years.

$\therefore P\left[1-\frac{1}{(1+r)^{t-x}}\right]=P_1[(1+r)^{x-t_1}-1]$.

$$\therefore\ x=\frac{\log(P+P_1)[(1+r)^t(1+r)^{t_1}]-\log[P(1+r)^{t_1}+P_1(1+r)^t]}{\log(1+r)}\ \ldots\ldots(3).$$

2nd Method.

$\frac{P}{(1+r)^t}$=present worth of P due t years hence.

$\frac{P_1}{(1+r)^{t_1}}$=present worth of P_1 due t_1 years hence.

$\frac{P+P_1}{(1+r)^x}$=present worth of $P+P_1$ due x years hence.

$\therefore$ Suppose $\frac{P}{(1+r)^t}+\frac{P_1}{(1+r)^{t_1}}=\frac{P+P_1}{(1+r)^x}$.

Then $x=\dfrac{\log(P+P_1)[(1+r)^t(1+r)^{t_1}-\log[P(1+r)^{t_1}+P_1(1+r)^t]}{\log(1+r)}$ ....(4).

But (4) and (3) being identical, either method may be used when compound interest is considered. The first, or correct, method by simple interest becomes very complicated when more than two payments are considered ; yet when we recall the fact that equation of payments is a subject of no practical importance, making approximate methods less desirous, it matters little how complicated the method may be if it is correct in theory.

The following method, which is found in most arithmetics is very often not much better than a good guess. A review of the solution will, at once, show the erroneousness of the method.

$P(t-x)r=$interest on P for $(t-x)$ years.

$P_1(x-t_1)r=$interest on P_1 for $(x-t_1)$ years.

$P(t-x)r=P_1(x-t_1)r$.

$\therefore x=\dfrac{Pt+P_1t_1}{P+P_1}$.

III. BY ANNUAL INTEREST.

$\dfrac{Pr[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}{1+r[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}=$discount on P for $(t-x)$ years.

$P_1r[(x-t_1)+\frac{1}{2}r(x-t_1)(x-t_1-1)]=$interest on P_1 for $(x-t_1)$ years.

$$\therefore \frac{Pr[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}{1+r[(t-x)+\frac{1}{2}r(t-x)(t-x-1)]}=P_1r[(x-t_1)+\tfrac{1}{2}r(x-t_1)(x-t_1-1)] \quad .....(5).$$

From (5) x, the equated time, can be found.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from the August-September Number.]

PROPOSITION XXVIII. *If two straights AX, BX (produced from any-sized straight AB toward the same parts, the first under an acute angle, and the other perpendicularly) mutually approach each other ever more without any certain limit, save at their infinite production ; I say all angles (Fig. 33.) at any points L, H, D of AX, from which are dropped to the straight BX perpendiculars LK, HK, DK,*